Covariant Treatment of Neutrino Spin (Flavour) Conversion in Matter under the Influence of Electromagnetic Fields

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*Abstract.*Within the recently proposed [1, 2] Lorentz invariant formalism for description of neutrino spin evolution in presence of an arbitrary electromagnetic fields effects of matter motion and polarization are considered.

In [1, 2] the Lorentz invariant formalism for neutrino motion in nonmoving and isotropic matter under the influence of an arbitrary configuration of electromagnetic fields have been developed. We have derived the neutrino spin evolution Hamiltonian that accounts not only for the transversal to the neutrino momentum components of electromagnetic field but also for the longitudinal components. With the using of the proposed Hamiltonian it is possible to consider neutrino spin precession in an arbitrary configuration of electromagnetic fields including those that contain strong longitudinal components. We have also considered the new types of resonances in the neutrino spin precession $\nu_L \leftrightarrow \nu_R$ that could appear when neutrinos propagate in matter under the influence of different electromagnetic field configurations.

In the studies [1, 2] of the neutrino spin evolution we have focused mainly on description of influence of different electromagnetic fields, while modeling the matter we confined ourselves to the most simple case of nonmoving and unpolarized matter. Now we should like go to further and to generalize our approach for the case of moving and polarized homogeneous matter.

To derive the equation for the neutrino spin evolution in electromagnetic field $F_{\mu\nu}$ in such a matter we again start from the Bargmann-Michel-Telegdi (BMT) equation [3] for the spin vector S^μ of a neutral particle that has the following form

$$\frac{dS^\mu}{d\tau} = 2\mu\big\{F^{\mu\nu}S_\nu - u^\mu(u_\nu F^{\nu\lambda}S_\lambda)\big\} + 2\epsilon\big\{\tilde{F}^{\mu\nu}S_\nu - u^\mu(u_\nu \tilde{F}^{\nu\lambda}S_\lambda)\big\}, \qquad (1)$$

This form of the BMT equation corresponds to the case of the particle moving with constant speed, $\vec{\beta} = const$, $(u_\mu = (\gamma, \gamma\vec{\beta}), \gamma = (1-\beta^2)^{-1/2})$, in presence of an electromagnetic field $F_{\mu\nu}$. Here μ is the fermion magnetic moment and $\tilde{F}_{\mu\nu}$ is the dual electromagnetic field tensor. The neutrino spin vector also satisfies the usual conditions, $S^2 = -1$ and $S^\mu u_\mu = 0$. Equation (1) covers also the case of a neutral fermion having static non-vanishing electric dipole moment, ϵ. Note that the term proportional to ϵ violates T invariance.

The BMT spin evolution equation (1) is derived in the frame of electrodynamics, the model which is P invariant. Our aim is to generalize this equation for the case when effects of various neutrino interactions (for example, weak interaction for which P invariance is broken) with moving and polarized matter are also taken into account. Effects of possible P nonconservation and non-trivial properties of matter (i.e., its motion and polarization) have to be reflected in the equation that describes the neutrino spin evolution in an electromagnetic field.

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The Lorentz invariant generalization of eq.(1) for our case can be obtained by the substitution of the electromagnetic field tensor $F_{\mu\nu} = (\vec{E}, \vec{B})$ in the following way:

$$F_{\mu\nu} \to F_{\mu\nu} + G_{\mu\nu}. \tag{2}$$

In evaluation of the tensor $G_{\mu\nu}$ we demand that the neutrino evolution equation has to be linear over the neutrino spin vector S_μ, electromagnetic field $F_{\mu\nu}$, and such characteristics of matter (which is composed of different fermions, $f = e,\ n,\ p...$) as fermions currents

$$j_f^\mu = (n_f, n_f \vec{v}_f), \tag{3}$$

and fermions polarizations

$$\lambda_f^\mu = \Big(n_f (\vec{\zeta}_f \vec{v}_f), n_f \vec{\zeta}_f \sqrt{1 - v_f^2} + \frac{n_f \vec{v}_f (\vec{\zeta}_f \vec{v}_f)}{1 + \sqrt{1 - v_f^2}} \Big). \tag{4}$$

Here n_f, $\vec{v}_f$, and $\vec{\zeta}_f$ $(0 \le |\vec{\zeta}_f|^2 \le 1)$ denote, respectively, the number density of the background fermions f, the speed of the reference frame in which the mean momentum of fermions f is zero, and the mean value of the polarization vectors of the background fermions f in the above mentioned reference frame. For each type of fermions f there are only three vectors, u_μ^f, j_μ^f, and λ_μ^f, using which the tensor $G_{\mu\nu}$ have to be constructed. If j_μ^f and λ_μ^f are slowly varying functions in space and time (this condition is similar to one imposed on the electromagnetic field tensor $F_{\mu\nu}$ in the derivation of the BMT equation) then one can construct only four tensors (for each of the fermions f) linear in respect to the characteristics of matter:

$$G_1^{\mu\nu} = \epsilon^{\mu\nu\rho\lambda} u_\lambda j_\rho, \quad G_2^{\mu\nu} = \epsilon^{\mu\nu\rho\lambda} u_\lambda \lambda_\rho, \tag{5}$$

$$G_3^{\mu\nu} = u^\mu j^\nu - j^\mu u^\nu, \quad G_4^{\mu\nu} = u^\mu \lambda^\nu - \lambda^\mu u^\nu. \tag{6}$$

Thus, in general case of neutrino interaction with different background fermions f we introduce for description of matter effects antisymmetric tensor

$$G^{\mu\nu} = \epsilon^{\mu\nu\rho\lambda} g_\rho^{(1)} u_\lambda - (g^{(2)\mu} u^\nu - u^\mu g^{(2)\nu}), \tag{7}$$

where

$$g^{(1)\mu} = \sum_f \rho_f^{(1)} j_f^\mu + \rho_f^{(2)} \lambda_f^\mu, \quad g^{(2)\mu} = \sum_f \xi_f^{(1)} j_f^\mu + \xi_f^{(2)} \lambda_f^\mu. \tag{8}$$

Summation is performed over fermions f of the background. The explicit expressions for the coefficients $\rho_f^{(1),(2)}$ and $\xi_f^{(1),(2)}$ could be found if the particular model of neutrino interaction is chosen. In the usual notations the antisymmetric tensor $G_{\mu\nu}$ can be written in the form,

$$G_{\mu\nu} = \big(-\vec{P},\ \vec{M}), \tag{9}$$

where

$$\vec{M} = \gamma \big\{ (g_0^{(1)} \vec{\beta} - \vec{g}^{(1)}) - [\vec{\beta} \times \vec{g}^{(2)}] \big\},\ \vec{P} = -\gamma \big\{ (g_0^{(2)} \vec{\beta} - \vec{g}^{(2)}) + [\vec{\beta} \times \vec{g}^{(1)}] \big\}. \tag{10}$$

It worth to note that the substitution (2) implies that the magnetic $\vec{B}$ and electric $\vec{E}$ fields are shifted by the vectors $\vec{M}$ and $\vec{P}$, respectively:

$$\vec{B} \to \vec{B} + \vec{M}, \quad \vec{E} \to \vec{E} - \vec{P}. \tag{11}$$

In the case of nonmoving, $\vec{v}_f = 0$, and unpolarized, $\vec{\zeta}_f = 0$, matter we get, confirming our previous result [1, 2],

$$G_{\mu\nu} = \Big(\gamma\vec{\beta}\sum_f \xi_f^{(1)} n_f, \gamma\vec{\beta}\sum_f \rho_f^{(1)} n_f\Big). \tag{12}$$

We finally arrive to the following equation for the evolution of the three-dimensional neutrino spin vector $\vec{S}$ accounting for the direct neutrino interaction with electromagnetic field $F_{\mu\nu}$ and matter (which is described by the tensor $G_{\mu\nu}$):

$$\frac{d\vec{S}}{dt} = \frac{2\mu}{\gamma}\Big[\vec{S}\times(\vec{B}_0+\vec{M}_0)\Big] + \frac{2\epsilon}{\gamma}\Big[\vec{S}\times(\vec{E}_0-\vec{P}_0)\Big]. \tag{13}$$

The derivative in the left-hand side of eq.(13) is taken with respect to time t in the laboratory frame, whereas the values $\vec{B}_0$ and $\vec{E}_0$ are the magnetic and electric fields in the neutrino rest frame given in terms of the transversal, $\vec{F}_\perp$, and longitudinal, $\vec{F}_\parallel$, in respect to the direction of the neutrino motion field $\vec{F} = \ \vec{B}, \vec{E}$ components in the laboratory frame,

$$\begin{gathered}\vec{B}_0 = \gamma\big(\vec{B}_\perp + \tfrac{1}{\gamma}\vec{B}_\parallel + \sqrt{1-\tfrac{1}{\gamma^2}}\big[\vec{E}_\perp\times\vec{n}\big]\big),\\ \vec{E}_0 = \gamma\big(\vec{E}_\perp + \tfrac{1}{\gamma}\vec{E}_\parallel - \sqrt{1-\tfrac{1}{\gamma^2}}\big[\vec{B}_\perp\times\vec{n}\big]\big), \vec{n} = \vec{\beta}/\beta.\end{gathered} \tag{14}$$

The influence of matter on the neutrino spin evolution in eq.(13) is given by the vectors $\vec{M}_0$ and $\vec{P}_0$ which in the rest frame of neutrino can be expressed in terms of quantities determined in the laboratory frame

$$\vec{M}_0 = \gamma\vec{\beta}\Big(g_0^{(1)} - \tfrac{\vec{\beta}\vec{g}^{(1)}}{1+\gamma^{-1}}\Big) - \vec{g}^{(1)}, \tag{15}$$

$$\vec{P}_0 = -\gamma\vec{\beta}\Big(g_0^{(2)} - \frac{\vec{\beta}\vec{g}^{(2)}}{1+\gamma^{-1}}\Big) + \vec{g}^{(2)}. \tag{16}$$

Let us determine the coefficients $\rho_f^{(i)}$ and $\xi_f^{(i)}$ in eq.(8) for the particular case of the electron neutrino propagation in moving and polarized electron gas. We consider the standard model of interaction supplied with $SU(2)$-singlet right-handed neutrino ν_R. The neutrino effective interaction Lagrangian reads

$$L_{eff} = -f^\mu\Big(\bar{\nu}\gamma_\mu\frac{1+\gamma^5}{2}\nu\Big), \tag{17}$$

where

$$f^\mu = \frac{G_F}{\sqrt{2}}\Big((1+4\sin^2\theta_W)j_e^\mu - \lambda_e^\mu\Big). \tag{18}$$

In this case neutrino electric dipole moment vanishes, $\epsilon = 0$, so that the coefficients $\xi_e^{(i)} = 0$, and from the obvious relation, $f_\mu = 2\mu g_\mu^{(1)}$, it follows

$$\rho_e^{(1)} = \frac{G_F}{2\mu\sqrt{2}}(1+4\sin^2\theta_W),\ \rho_e^{(2)} = -\frac{G_F}{2\mu\sqrt{2}}. \tag{19}$$

If for the neutrino magnetic moment we take the vacuum one-loop contribution [4,5]

$$\mu_\nu = \frac{3}{8\sqrt{2}\pi^2} e G_F m_\nu,$$

then

$$\rho^{(1)} = \frac{4\pi^2}{3em_\nu}(1 + 4\sin^2\theta_W), \quad \rho^{(2)} = -\frac{4\pi^2}{3em_\nu}.$$

We should like to note that solutions of the derived eq.(13) for the neutrino spin evolution in moving and polarized matter and, correspondingly, the neutrino oscillation probabilities and effective mixing angles θ_{eff} can be obtained for different configurations of electromagnetic fields in a way similar to that described in [1,2,6].

Let us consider the case of neutrino propagating in the relativistic flux of electrons. Using expressions for the vector $\vec{M}_0$, eqs. (8), (15), we find,

$$\begin{gathered}\vec{M}_0 = n_e\gamma\vec{\beta}\Big\{\big(\rho^{(1)} + \rho^{(2)}\vec{\zeta}_e\vec{v}_e\big)(1 - \vec{\beta}\vec{v}_e) + \\ +\rho^{(2)}\sqrt{1-v_e^2}\Big[\tfrac{(\vec{\zeta}_e\vec{v}_e)(\vec{\beta}\vec{v}_e)}{1+\sqrt{1-v_e^2}} - \vec{\zeta}_e\vec{\beta}\Big] + O(\gamma^{-1})\Big\}.\end{gathered} \tag{20}$$

In the case of slowly moving matter, $v_e \ll 1$, we get

$$\vec{M}_0 = n_e\gamma\vec{\beta}\Big(\rho^{(1)} - \rho^{(2)}\vec{\zeta}_e\vec{\beta}\Big). \tag{21}$$

For the unpolarized matter eq.(21) reproduces the Wolfenstein term and confirms our previous result [1,2]. In the opposite case of relativistic flux, $v_e \sim 1$, we find,

$$\vec{M}_0 = n_e\gamma\vec{\beta}\Big(\rho^{(1)} + \rho^{(2)}\vec{\zeta}_e\vec{v}_e\Big)\Big(1 - \vec{\beta}\vec{v}_e\Big). \tag{22}$$

If we introduce the invariant electron number density,

$$n_0 = n_e\sqrt{1 - v_e^2}, \tag{23}$$

then it follows,

$$\vec{M}_0 = n_o\gamma\vec{\beta}\tfrac{(1-\vec{\beta}\vec{v}_e)}{\sqrt{1-v_e^2}}\Big(\rho^{(1)} + \rho^{(2)}\vec{\zeta}_e\vec{v}_e\Big). \tag{24}$$

Thus in the case of the parallel motion of neutrinos and electrons of the flux matter effect contribution to the neutrino spin evolution equation (13) is suppressed. It should be noted that this fenomenon can exist also for neutrino flavour oscillations in moving matter.

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